Geodesy 1B (GED209)
Lecture No: 7

Strength of Figure

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Establishment of Surveying Controls

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- Astronomical positioning
- Global Positioning System (GPS)

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Triangulation

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Definition

- The process of a measuring system comprised of connected triangles whose vertices are stations marked on the surface of the earth and in which angular observations are supported by occasional distance and astronomical observation.

Purpose

- To establish the accurate control points for plane and geodetic surveys of large areas
- To establish the accurate control points for photogrammetric surveys
- Accurate location of engineering works

Principle of Triangulation

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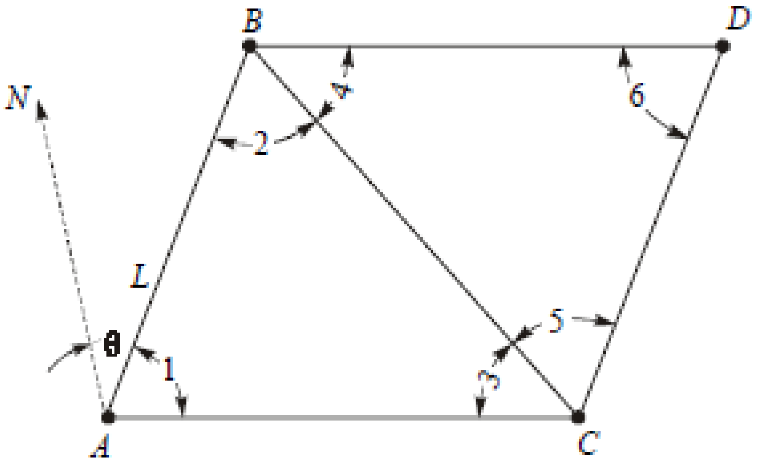
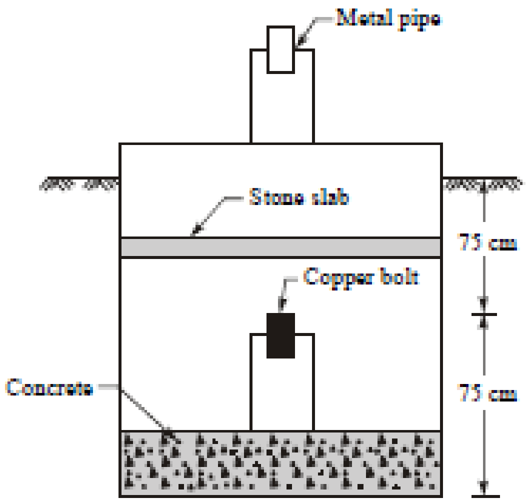
- Entire area to be surveyed is converted into framework of triangles
- If the length and bearing of one side and three angles of a triangle are measured precisely, the lengths and directions of other two sides can be computed.
- Precisely measured line is called **base line**.
- Computed two lines are used as base lines for two interconnected triangles.
- Vertices of the individual triangles are known as **triangulation stations**.
- Extending this process network of triangles can be computed over the entire area.
- As a **check** the length of one side of last triangle is also measured and compared with the computed one.
- Subsidiary bases are measured at suitable **intervals** to minimize accumulation of **errors** in lengths.
- Astronomical observations are made at intermediate stations to control the error in azimuth.
- Those triangulation stations are called **Laplace Stations**.



Criteria for selection of stations

Triangulation stations must be selected carefully to save a lot of time and funds by keeping the following key points in mind:



- Triangulation stations should be intervisible.
- Stations should be easily accessible with instruments.
- Station should form well-conditioned triangles.
- Stations should be located so that the survey lines are neither too small nor too long
- Cost of clearing and cutting and building towers should be minimum
- No line of sight should pass over the industrial areas to avoid irregular atmospheric refraction

Factors to be considered in selecting a figure

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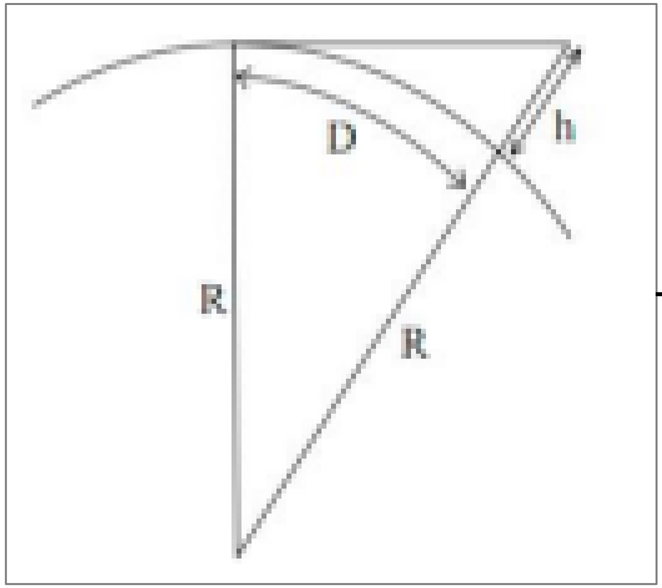
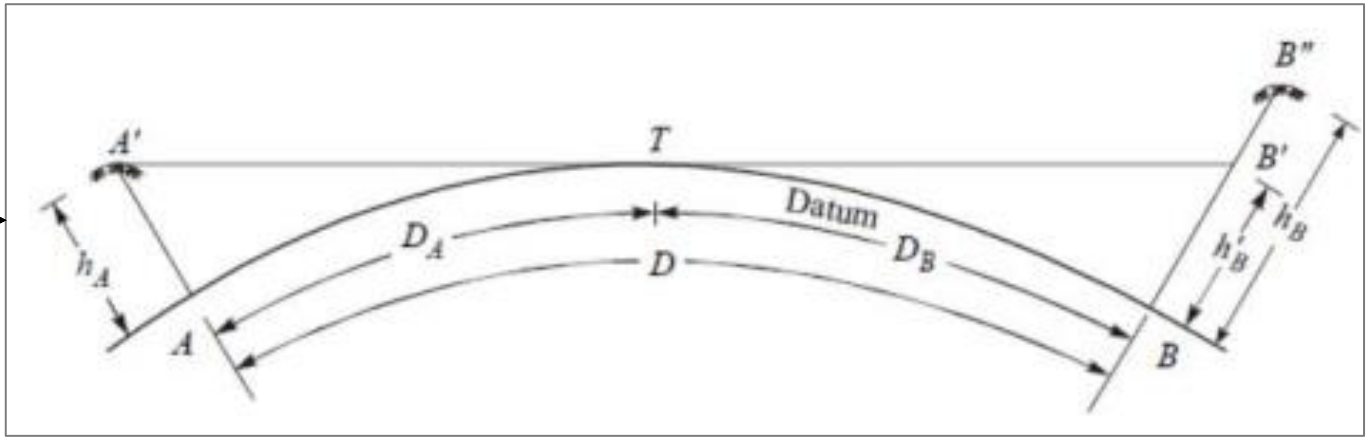
- Simple triangles should be preferably equilateral.
- Braced quadrilaterals should be preferably squares.
- Centered polygons should be regular.
- No angle of the figure, opposite a known side should be small.
- The triangles, whose angles are less than 30° or more than 120° should be avoided in the chain of triangles.
- In case of quadrilaterals no angle should be less than 30° or greater than 150°.
- The sides of the figures should be of comparable length.

Intervisibility between stations

- Does the intervening ground obstruct the intervisibility?
- The distance of horizon from the station of known elevation is given by:

$$h = \frac{D^2}{2R}(1 - 2k)$$





Strength of Figure

- A measure of the judicious selection of the framework consisting of triangles and quadliterals employed for triangulation.
- Determination of the figure which gives the least error in calculated length of last line in the system due to the shape of triangles and computation of the figures.

$$R = F\sum(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$$

R: Strength of Figure

F: Strength of Figure Factor F = $\frac{D-C}{D}$

$\delta_A\delta_B$: The logarithmic differences corresponding to 1" for distance angles A and B

Strength of Figure

- **F: Strength of Figure Factor F =** $\boldsymbol{\frac{D-C}{D}}$

D: the total number of observed directions except the base line

C: the total number of conditions $C = (L' - S' + 1) + (L - 2S + 3)$

L': number of lines observed in both directions including the baseline.

S': number of occupied stations

L: total number of lines

S: total number of stations

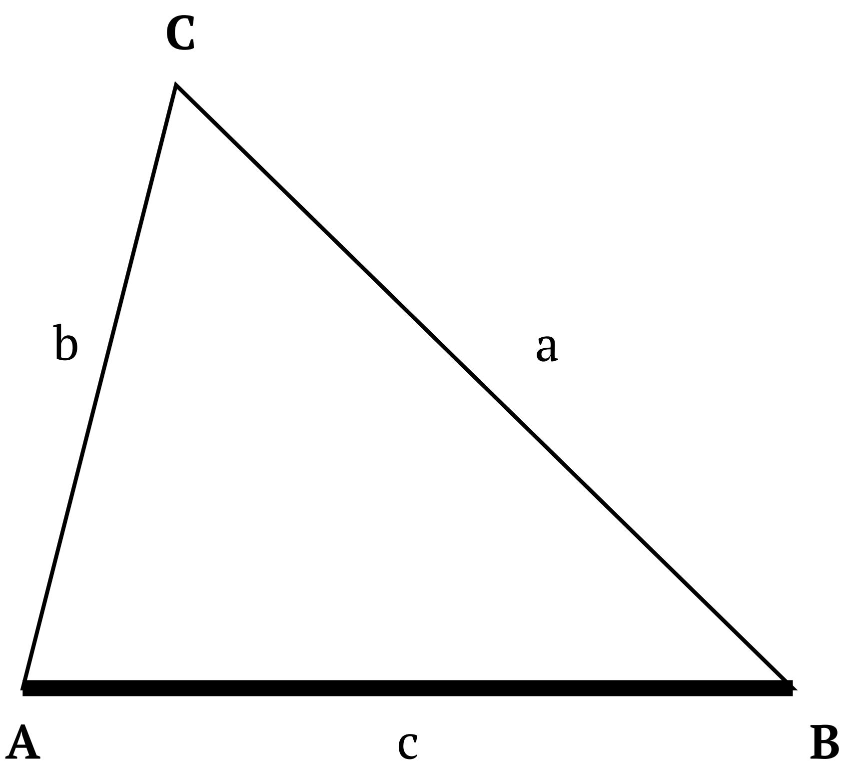
Strength of Figure, Why?

➢ In a triangulation network, all angles are observed and a base line while the lengths of other lines are computed based on sine rule.

$$\frac{AB}{\sin C} = \frac{BC}{\sin A} = \frac{AC}{\sin B}$$

$$\therefore BC = \frac{AB \sin A}{\sin C}$$

$$\therefore AC = \frac{AB \sin B}{\sin C}$$

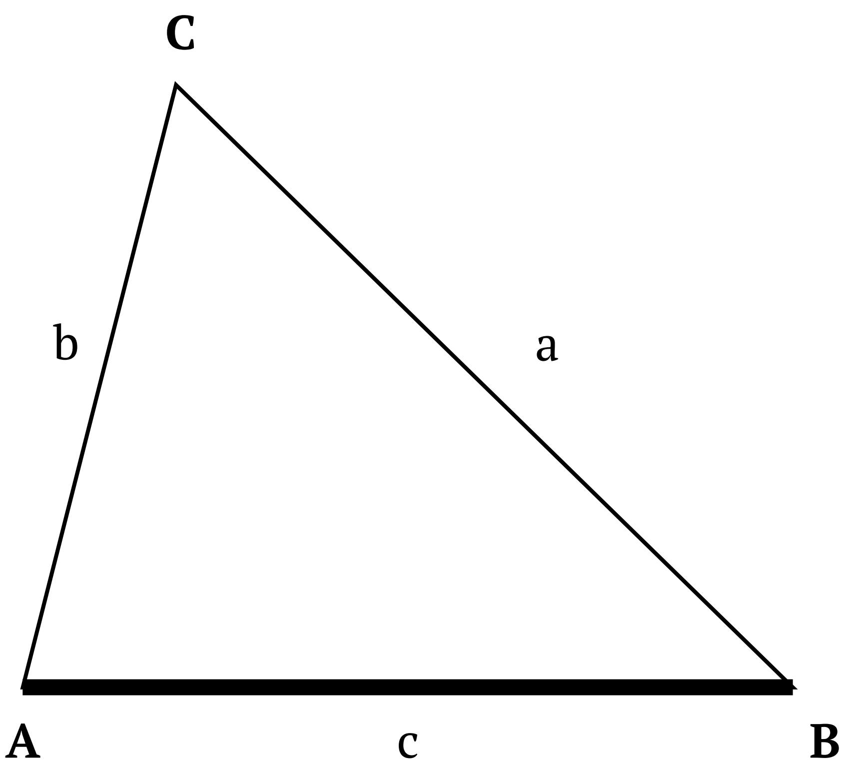


➢ How much a side length is affected if an angle contain an error of 1 arcsecond?

Strength of Figure, Why?

➢ How much a side length is affected if an angle contain an error of 1 arcsecond?

$$\mathrm{Difference} = \log \sin A - \log \sin(A + 1'') = 2.1 \cot A$$



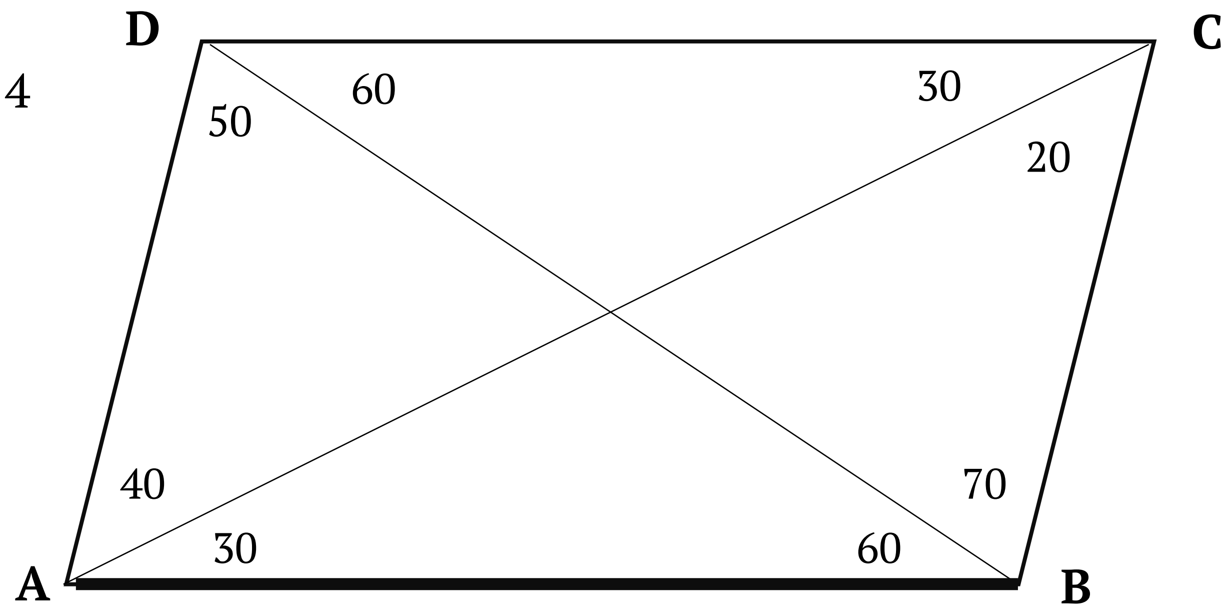
➢ **Determine the best route to calculate the side CD from the known side AB in the shown figure.**

$L' = 6, L = 6, S = 4\ , and\ S' = 4$

$C = (L' - S' + 1) + (L - 2S + 3)$ = (6 – 4 + 1) + (6 – 8 +3) = 4

$D = 10$

$$\boldsymbol{F = \frac{D - C}{D} = \frac{10 - 4}{10} = 0.60}$$



Numerical Examples

Route No	Triangle	Known Side	Unknown Side	Distance Angle		$(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	$\sum(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	R
				A	B			
R_1	ABC	AB	BC	20	30	67.505	70.49	42.29
	BCD	BC	CD	60	70	2.98		
R_2	ABD	AB	BD	50	70	5.036	10.07	6.04
	BDC	BD	DC	50	70	5.036		
R_3	BAC	BA	AC	20	130	26.227	31.16	18.7
	ACD	AC	CD	110	40	4.934		
R_4	BAD	BA	AD	50	60	6.711	35.31	21.18
	ADC	AD	DC	30	40	28.596		

Then, the best route to compute side CD from the baseline AB is $\mathbf{R_2}$ using the triangles **ABD** and **BDC**.

Numerical Examples

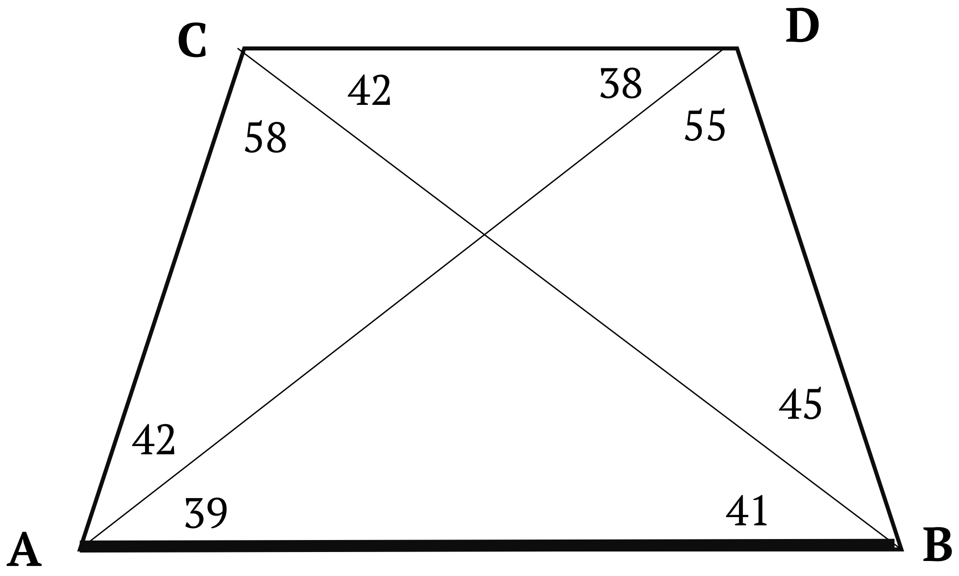
- Determine the best route to calculate the side CD from the known side AB in the shown figure.

$L' = 6, L = 6, S = 4 \text{ , } and\ S' = 4$

$C = (L' - S' + 1) + (L - 2S + 3) = (6 - 4 + 1) + (6 - 8 + 3) = 4$

$D = 10$

$$\boldsymbol{F = \frac{D - C}{D} = \frac{10 - 4}{10} = 0.60}$$



Numerical Examples

Route No	Triangle	Known Side	Unknown Side	Distance Angle		$(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	$\sum(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	R
				A	B			
R_1	ABC	AB	BC	58	81	2.27	6.46	3.87
	BCD	BC	CD	93	45	4.19		
R_2	ABD	AB	BD	55	39	12.7	27.45	16.47
	BDC	BD	DC	42	45	14.47		
R_3	BAC	BA	AC	58	41	10.73	30.12	18.07
	ACD	AC	CD	38	42	19.39		
R_4	BAD	BA	AD	55	86	2.4	7.11	4.27
	ADC	AD	DC	100	42	4.7		

Then, the best route to compute side CD from the baseline AB is $\mathbf{R_1}$ using the triangles **ABC** and **BCD**.

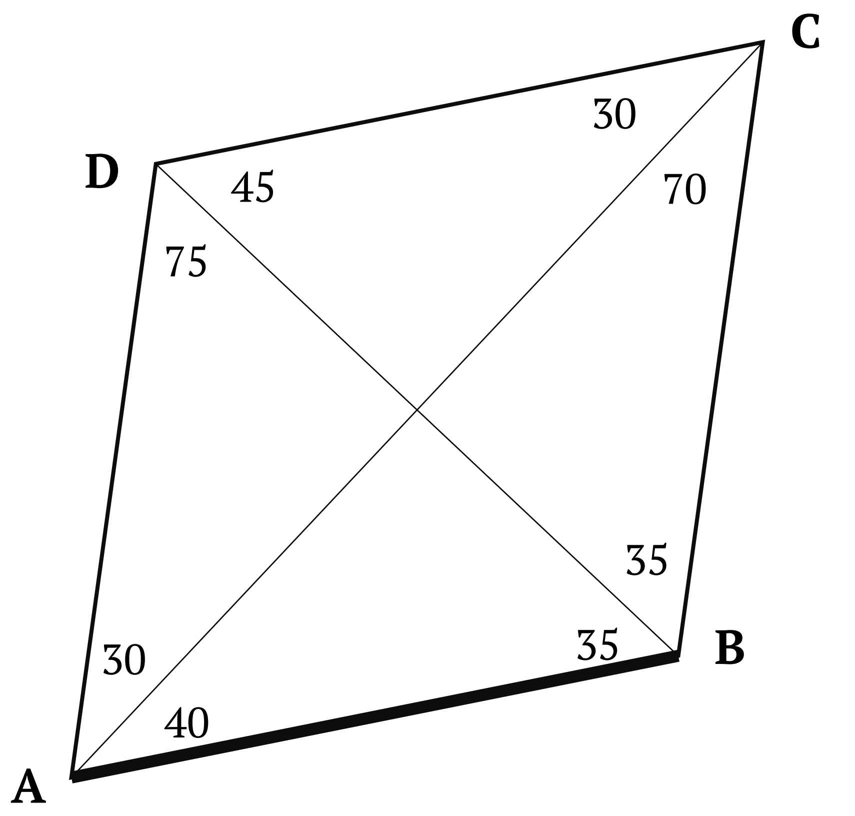
- Determine the best route to calculate the side CD from the known side AB in the shown figure.

$L' = 6, L = 6, S = 4\ , and\ S' = 4$

$C = (L' - S' + 1) + (L - 2S + 3) = (6 - 4 + 1) + (6 - 8 + 3) = 4$

$D = 10$

$$\boldsymbol{F = \frac{D - C}{D} = \frac{10 - 4}{10} = 0.60}$$



Numerical Examples

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Route No	Triangle	Known Side	Unknown Side	Distance Angle A	Distance Angle B	$(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	$\sum(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	R
R_1	ABC	AB	BC	70	40	8.76	28.46	17.07
	BCD	BC	CD	45	35	19.70		
R_2	BAC	BA	AC	70	70	67.16	77.45	46.47
	ACD	AC	DC	120	30	10.29		
R_3	ABD	AB	BD	75	70	1.33	9.35	5.61
	BDC	BD	CD	100	35	8.02		
R_4	BAD	BA	AD	75	35	10.99	50.68	30.40
	ADC	AD	DC	30	30	39.69		

Then, the best route to compute side CD from the baseline AB is $\mathbf{R_3}$ using the triangles **ABD** and **BDC**.

Strength of Figure

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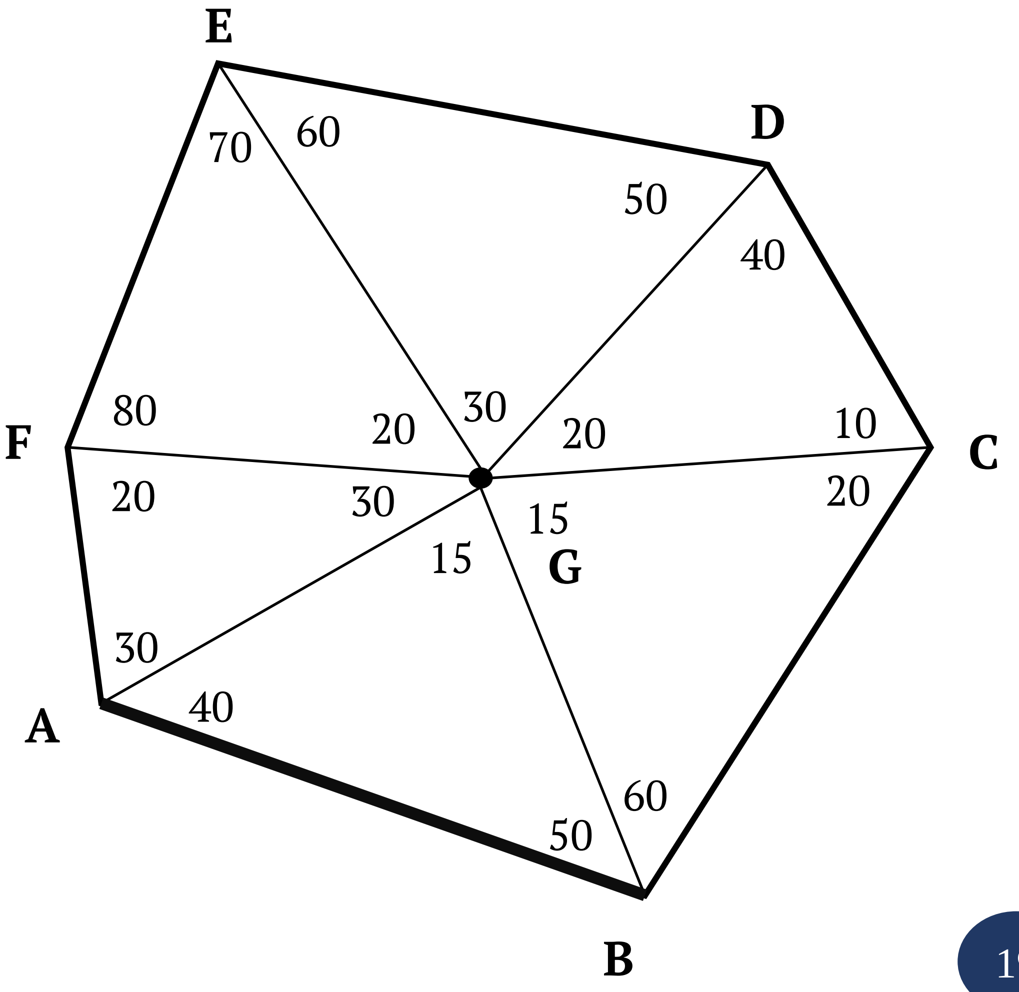
- Determine the best route to calculate the side ED from the known side AB in the shown figure.

$L' = 12, L = 12, S = 7\ , and\ S' = 7$

$C = (L' - S' + 1) + (L - 2S + 3) = (12 – 7+ 1) + (12 – 14 +3) = 7$

$D = 22$

$$\boldsymbol{F = \frac{D - C}{D} = \frac{22 - 7}{22} = 0.68}$$



Strength of Figure

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Route No	Triangle	Known Side	Unknown Side	Distance Angle		$(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	$\sum(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	R
				A	B			
R_1	ABG	AB	BG	15	40	87.3	326.1×0.68	221.7
	GBC	BG	GC	20	60	41.8		
	GCD	GC	GD	40	10	177.9		
	GDE	GD	ED	60	30	19.1		
R_2	ABG	AB	AG	15	50	78.3	169.6×0.68	115.3
	AGF	AG	GF	20	30	67.5		
	GFE	GF	EG	70	80	1.0		
	EGD	EG	ED	50	30	22.7		

Then, the best route to compute side ED from the baseline AB is $\mathbf{R_2}$ using the triangles **ABG, AGF, GFE**, and **EGD**

Strength of Figure

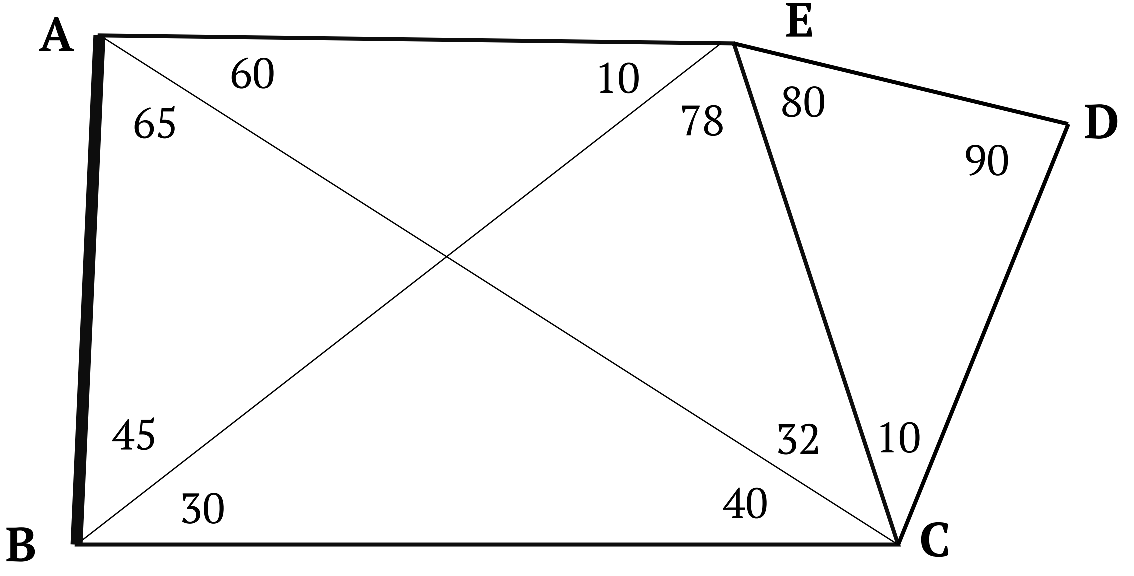
➢ **Determine the best route to calculate the side ED from the known side AB in the shown figure.**

$L' = 8, L = 8, S = 5\,, and\ S' = 5$

$C = (L' - S' + 1) + (L - 2S + 3)$ = (8 – 5+ 1) + (8 – 10 +3) = 5

$D = 14$

$$\boldsymbol{F = \frac{D - C}{D} = \frac{14 - 5}{14} = 0.64}$$



Strength of Figure

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Route No	Triangle	Known Side	Unknown Side	Distance Angle		$(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	$\sum(\delta_A^2 + \delta_A\delta_B + \delta_B^2)$	R
				A	B			
R_1	ABC	AB	BC	40	65	9.7	166.6	106.6
	BCE	BC	EC	78	30	15.1		
	CED	EC	ED	90	10	141.85		
R_2	ABC	AB	AC	40	75	7.98	151.4	96.89
	ACE	AC	EC	83	60	1.6		
	CED	EC	ED	90	10	141.85		
R_3	ABE	AB	AE	10	45	171.3	329.95	211.2
	AEC	AE	EC	32	60	16.8		
	CED	EC	ED	90	10	141.8		
R_4	ABE	AB	BE	10	125	126.5	284.5	182.1
	BEC	BE	EC	72	30	16.18		
	CED	EC	ED	90	10	141.85		

Then, the best route to compute side ED from the baseline AB is **R_2** using the triangles **ABC, ACE,** and **CED**

End of Presentation

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THANK YOU